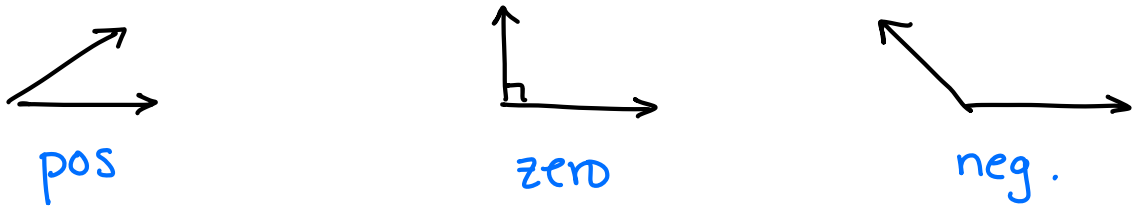
Line Integrals of Vector Fields

Intuitive Intro to Line Integral of V.F.

Dot product tells how much 2 vectors "cooperate"



Line integral is a continuously accumulating version of the dot product.

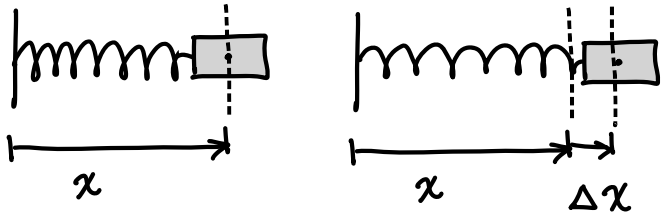
Application : Work

- 1D, const force : $W = Fd$

- 1D, force varies w/ position

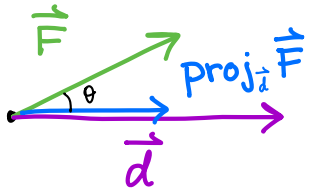
$$\Delta W \approx F(x)\,\Delta x$$

$$W = \int_a^b F(x)\,dx$$

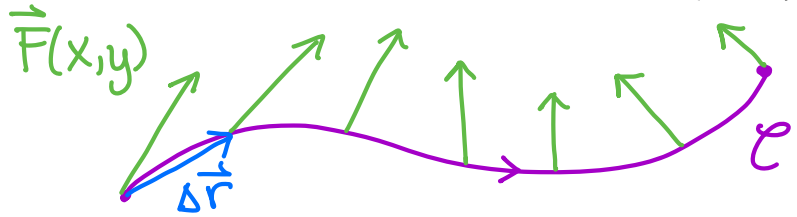


- 2D, 3D, motion in a straight line
  const force, not alligned w/ displacement

$$W = \vec{F} \cdot \vec{d}$$
$$= \|\vec{F}\|\,\|\vec{d}\|\cos\theta$$
$$= \pm \|\text{proj}_{\vec{d}}\vec{F}\| \cdot \|\vec{d}\|$$



- 2D, 3D, motion along a curve & force varies w/ position

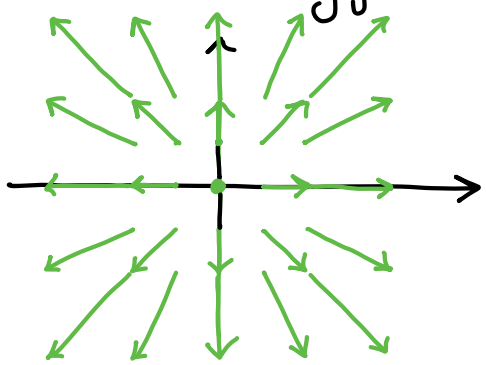


$$\Delta W \approx \vec{F}(x,y) \cdot \Delta\vec{r}$$

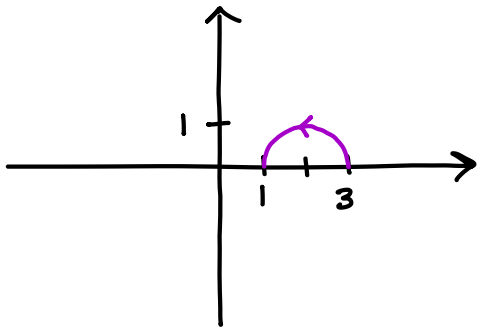
$$W = \int_{\mathcal{C}} \vec{F} \cdot d\vec{r} \quad \leftarrow \text{Line Integral}$$

Example:

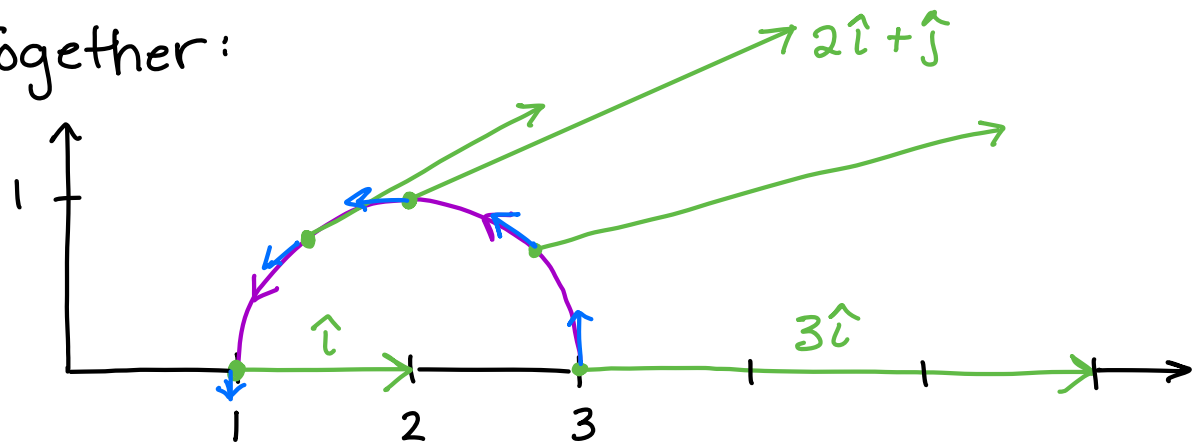
$\vec{F} = x\hat{\imath} + y\hat{\jmath}$



Curve $\mathcal{C}$: semicircle



Together:



Expect line integral to be pos., neg., or zero?